Coronal Solar Magnetism Observatory
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Some Considerations for a High Etendue Birefringent Filter

Steven Tomczyk
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Summary: Observing an extended object like the solar corona with a large aperture coronagraph places a severe requirement on the etendue of the spectrometer that is employed. We investigate the etendue requirement for a wide-field birefringent filter following a 1.5 m aperture coronagraph. We find that a filter constructed with $LiNbO_3$ can meet the etendue requirement.

1) COSMO Etendue Requirements

The design goals for a filter behind the COSMO large coronagraph are:

1° full field-of-view (FOV)
1.5 m telescope aperture
0.14 nm FWHM filter bandpass at 1074.7 nm

The etendue (also called luminosity or throughput), is a constant of the system and is given by the product of the solid angle and the area. We can estimate the etendue at the aperture of the telescope, and require that the etendue of the filter is equal to or greater than the etendue at the aperture. If it is not, then the light gathering power of the telescope will be compromised. The solid angle accepted by the telescope is:

$$\Omega = 4\pi \sin^2 \frac{\theta}{2} \ \text{sr} \qquad [1]$$

where θ is the half-angle = 0.5°, then $\Omega = 2.392 \cdot 10^{-4}$ sr. The area of the telescope aperture is $A = 1.767 \cdot 10^4$ cm^2. Then, the required etendue is:

$$L = 4.23 \text{ cm}^2 \text{ sr}$$

For comparison, the etendue of the 4-m ATST telescope with a 5 arc minute full FOV is 0.21 cm^2 sr. This means that the light gathering power of the COSMO coronagraph will be a factor of 20 times greater than the ATST.

2) Etendue of Wide-Field Birefringent Filters

For a wide-field birefringent filter, the fractional wavelength shift as a function of the angle through the filter away from the normal, θ, is given to second order by[1]:

$$\frac{\Delta\lambda}{\lambda} = -\frac{1}{4n_o^2}\left(\frac{n_e - n_o}{n_e}\right)\sin^2\theta \qquad [2]$$

where n_e and n_o are the extraordinary and ordinary indices of the material, respectively. For the sake of simplicity, we will denote the factor in front of the $\sin^2\theta$ term as F. Clearly, a small value of F corresponds to a filter with a lower angular sensitivity. This occurs for high index, low birefringence materials.

Assuming small angles, so that $\sin^2 x \approx x^2$, and using the definition of spectral resolution, $R=\lambda/\Delta\lambda$, we can combine equations [1] and [2] and derive an equation for the etendue of the filter:

$$L = \frac{\pi A}{RF} \qquad [3]$$

Setting $\Delta\lambda$ equal to half of the FWHM of the filter, 0.07 nm, gives a value of the spectral resolution of $1.54 \cdot 10^4$. Using λ = 1074.7 nm and the etendue requirement of 4.23 cm^2 sr sets a requirement on the ratio A/F for the filter to be:

$$\frac{A}{F} = \frac{LR}{\pi} \geq 2.1 \cdot 10^4 \text{ cm}^2$$

The question is whether this ratio is achievable with available crystals. To evaluate this, I have searched the web for information on crystals which is summarized in Table 1. The diameter listed is the maximum diameter I could find; my search was not especially extensive and larger diameters may be available for some crystals.

Table 1

Crystal	n_o	n_e	n_e-n_o	F	Diameter (mm)	A/F (cm^2)	Length (mm)
MgF_2	1.384	1.396	+0.012	1.12e-3	120	1.0e5	605
$LiNbO_3$	2.286	2.203	-0.086	1.87e-3	100	4.2e4	84
SiO_2 (quartz)	1.543	1.552	+0.009	6.09e-4	50	3.2e4	807
$PbMoO_4$	2.386	2.262	-0.124	2.41e-3	60	1.2e4	59
TeO_2	2.260	2.142	-0.118	2.70e-3	50	7.3e3	62
BaB_2O_4	1.658	1.584	-0.073	4.19e-3	50	4.6e3	99
YVO_4	1.993	2.215	+0.222	6.31e-3	38	1.8e3	33
$CaCO_3$ (calcite)	1.656	1.485	-0.171	1.05e-2	40	1.2e3	42
TiO_2 (rutile)	2.583	2.865	+0.282	3.69e-3	25	1.1e3	26

The crystals are arranged in decreasing order of A/F, with the computation of F neglecting sign. The top three crystals in the table meet the COSMO requirement by virtue of their available diameters and low value of the factor F. The rightmost column shows the total length of crystal, *TL*, required to create a FWHM of 0.14 nm at 1074.7 nm computed by adapting the approximate equation relating filter FWHM and thickness[1]:

$$TL \cong \frac{0.88\lambda^2}{FWHM\left|n_e - n_o\right|}$$

The length calculation shows that the birefringence of quartz and MgF_2 are so low that birefringent filters made with them would be prohibitively long. This analysis suggests that $LiNbO_3$ is the material of choice for high etendue wide-field birefringent filters. The use of $LiNbO_3$ also presents the possibility of exploiting the electro-optical properties of that material to tune the filter.

3) Prefilter Etendue Considerations

For an interference filter, the wavelength shift as a function of the angle through the filter away from the normal is given by:

$$\frac{\Delta\lambda}{\lambda} = -\frac{1}{2n^2}\sin^2\theta$$

where *n* is the refractive index of the filter. This equation also holds for classical Fabry-Perot and Michelson interferometers. The refractive index for interference filters typically ranges between 1.5 and 2. Assuming an index of 2 results in an effective F factor for the prefilter of -0.125. Comparison with the F factors in the Table shows that interference filters are 2-3 orders of magnitude more sensitive to off axis light that wide-field birefringent filters. However, the prefilter is employed to block the unwanted orders of the birefringent filter and so will have a much wider bandpass than the birefringent filter itself. If individual blockers are used to separate the COSMO wavelength regions (1074.7, 1079.8 and 1083.0 nm) then the prefilter needs to have a FWHM bandwidth of about half the separation between the 1079.8 and 1083.0 nm regions, or about 1.6 nm. Setting $\Delta\lambda$ equal to half of this FWHM (0.8 nm) as above, we compute the spectral resolution of 1344. Combining this with F=.125 and L=4.23 cm^2 sr and equation [3] results in an area requirement for the beam going through the prefilter of:

$$A = \frac{RFL}{\pi} = 226.2 \text{ cm}^2$$

This corresponds to a prefilter diameter of at least 170 mm. Filters of this size would be difficult to obtain.

A more feasible alternative may be to increase the number of stages in the birefringent filter such that the filter can select between the three wavelength regions and only one prefilter could be used for the entire 1074.7-1083.0 nm region. This would allow an increase in the bandwidth of the prefilter to something like 20 nm, or a resolution of R=107, which would lower the diameter requirement to about 43 mm. Modeling indicates that a 6 stage birefringent filter would be consistent with a prefilter of this bandwidth.

References

1. Title and Rosenberg, 1981, Opt Eng, Vol. 20, 815.